

Running in the horizontal plane with a multi-modal dynamical robot

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Abstract—As the use of mobile robots expands to more diverse and challenging environments, improved mobility methods are required to provide these platforms with the ability to reliably negotiate these terrains. In this paper, we investigate SCARAB, a quadrupedal platform designed to rapidly traverse level, vertical, and inclined surfaces. This study extends previous work in which SCARAB's climbing performance was analyzed to demonstrate its ability to run effectively on level ground. We detail several modifications made to the platform to improve its capacity to both climb and run. This updated platform was used to investigate the influence of leg configuration and leg phasing on running performance and shows that SCARAB is able to run at speeds upward of 40cm s^{-1} . Furthermore, analysis of the dynamic characteristics shows similarity between those of the robot and the Lateral Leg Spring model, particularly for configurations that produce the highest running speeds. These results make SCARAB the first platform to effectively demonstrate horizontal plane dynamics quantitatively similar to this model, as well as the first to utilize two distinct dynamical locomotion modalities.

I. INTRODUCTION

Recently, legged locomotion has gained traction as an effective means of mobility for robotic platforms [1]–[4]. This interest has stemmed from the inherent advantage of legs in traversing unstructured terrains due to the use of isolated footholds, the ability to overcome large obstacles, and the capacity to adapt to uneven terrain [5]. In addition, significant progress has been made towards improving the speed, efficiency, and stability at which legged platforms can operate [6]–[8].

An additional advantage of utilizing legs is the potential to employ various locomotion modalities when presented with mobility challenges. Observation of legged animals shows that many are able to move in multiple regimes, using their legs in different ways as the environment changes [9]–[12]. For example, squirrels are able to run across the ground, bound up trees, and leap between limbs using the same appendages but in different manners. Some robots have already demonstrated some degree of multi-modal capabilities, such as RiSE [13], which can climb vertical surfaces and walk across level ground, albeit slowly.

To advance our understanding of the dynamics that lead to effective, high-speed legged robots, researchers have examined the dynamics of animal locomotion. Such studies have led to the formulation of reduced-order models that encapsu-

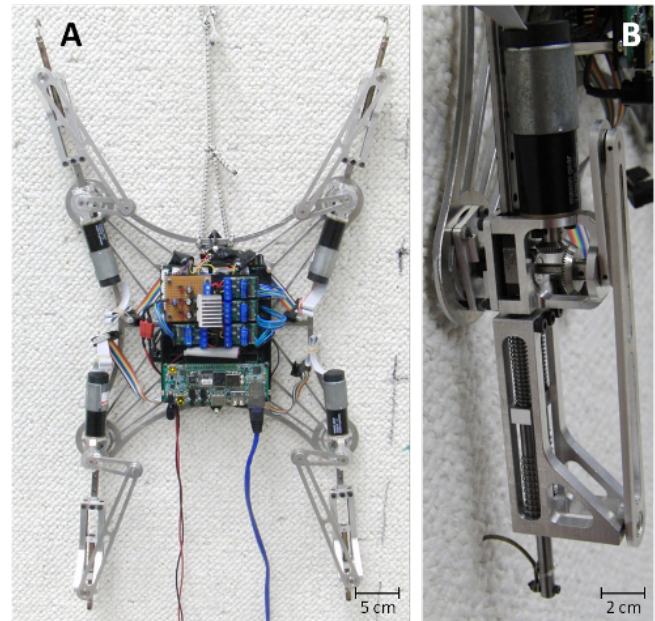


Fig. 1. (A) Photograph of the first version of SCARAB. The robot is on a carpeted vertical wall, which it is able to climb at speeds of up to 20cm s^{-1} . (B) Rear leg of the robot. The leg is actuated via a crank-slider mechanism to modulate the rest leg length. The leg also demonstrates differing compliance when in compression and extension due to the novel spring housing. Adapted from [19].

late the simplified body dynamics [14]. Various legged locomotion modalities have been modeled in this way, including walking [15], running [16], [17], and climbing [18]. Using these models as a template for dynamical legged locomotion, researchers have developed robots that are capable of emulating animal-like behaviors to move quickly and safely through environments via particular modalities.

In this paper, we examine SCARAB, shown in Fig. 1A, a legged robotic platform developed for high-speed locomotion in terrestrial and scansorial regimes [19]. The design of this platform was motivated by the Full-Goldman (FG) model of dynamic climbing [18] and the Lateral Leg Spring (LLS) model for horizontal plane running [17]. These two models possess many similarities that are likely to prove conducive to incorporating both behaviors in a robotic platform. Furthermore, recent work has shown that animals can exhibit

behavior similar to either model depending on the incline of the surface being traversed [20].

In a previous study [19], we characterized the behavior of SCARAB when climbing a vertical substrate; in this work, we extend the analysis of this platform to examine the dynamics when running on level ground. We investigate SCARAB's capacity for dynamic horizontal plane running and compare the locomotion dynamics to those exhibited by the LLS model to show that SCARAB is capable of moving in a qualitatively similar manner. We demonstrate that when leg orientation and phasing are properly selected, the platform is capable of running at speeds of greater than 40cm s^{-1} and the similarity to the LLS dynamics is improved.

In Section II, we discuss the reduced-order models for level ground running relevant to this study. Section III follows with an introduction to SCARAB platform and a description of the modifications made to facilitate running on level and inclined surfaces. The experimental procedures and analysis techniques are then presented in Section IV. This is followed in Section V by the results of the experiments and a discussion thereof. Section VI concludes the paper with an overview of the studies contributions and a reflection on avenues for future study.

II. BIOLOGICAL MODELS OF DYNAMIC RUNNING

Reduced-order models of dynamic locomotion serve as valuable templates for the design and control of legged robotic platforms. This due in large part to the complexity of biological legged locomotion, which does not lend itself to kinematic analyses and is often highly parameter dependent. However, dynamic biological behaviors can be distilled from redundant, high degree of freedom problems to simplified, reduced-order models that may only require the definition of a small set of parameters. For running systems, several models have been developed to capture the locomotion dynamics on level ground. Here, we review one such model, the Lateral Leg Spring model, which describes the horizontal plane dynamics of biological runners and was used as inspiration for the SCARAB platform.

The dynamics of running are commonly associated with motion in the sagittal plane, which has led the proliferation of the Spring Loaded Inverted Pendulum (SLIP) model as a template for running. While the sagittal plane dynamics of biological runners have been widely observed to match this model [21], SLIP only captures the behavior in two dimensions, constraining the system from exhibiting any side-to-side motion. While this assumption may suffice for animals with an upright posture, small runners with splayed legs, such as insects and amphibians, tend to generate significant lateral forces and motions when running [22]. To capture the horizontal plane dynamics, the Lateral Leg Spring model was developed [17]. The model consists of a body with a mass of m and a moment of inertia I that is attached to two axially elastic, transversely rigid legs with a stiffness k and a nominal rest length l_0 . The legs are attached to the body at a distance

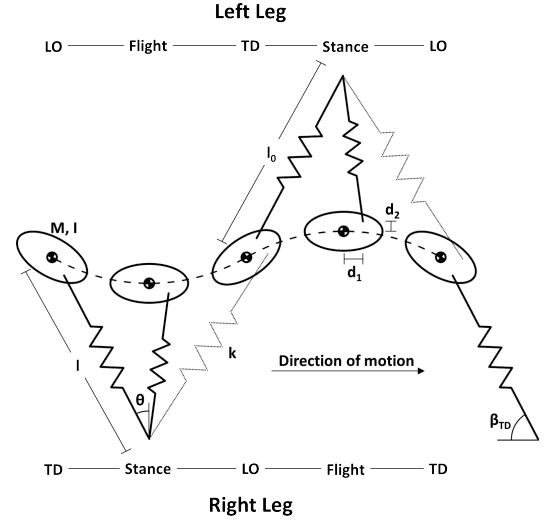


Fig. 2. Trajectory of the LLS model for a single stride. The system has a body of mass m and moment of inertia I attached to a spring of stiffness k and rest length l_0 , which is offset d_1 vertically and d_2 horizontally from the center of mass of the body. The trajectory begins with the stance phase of the right leg. The spring redirects the motion of the system, rotating the body as it is accelerated forward and to the right until the spring fully decompresses. Stance then switches to the left leg and the same behavior is shown, but mirrored. During each legs stance phase, the governing equations are written in terms of the polar coordinates l and θ , with the origin placed at the foot pivot.

d_1 from center of mass along the body centerline and d_2 to either side of the centerline via a freely rotating pin joint.

The dynamical behavior of LLS is modeled as a hybrid dynamic system. The hybrid nature of the system arises from steps being alternately taken by the left and right legs. To describe the progression of a stride, we will begin with a right step. The stride begins with the body moving towards the right side and the right leg establishing contact with the ground, which is modeled as a freely rotating pin joint between with distal end of the leg and the ground. At this point, the left leg, which is in flight, is rotated to the desired touchdown angle β_{TD} for the next step. The right leg compresses as the body continues moving towards it, storing the kinetic energy as elastic potential in the spring and redirecting the motion away from the leg contact. When the elastic energy is returned to the body and the force in the leg spring drops to zero, the contact between the right leg and the ground is broken while coincidentally contact between the left leg and ground is established. The same behavior for the left step as for the right. This pattern is repeated to produce the horizontal plane running. A sample of the trajectory generated through running with the LLS model is shown in Fig. 2. Although energetically conservative, the dynamics of LLS exhibit self-stabilizing behaviors for non-energetic perturbations when properly tuned and have been found to match the motion of running insects [23].

III. PHYSICAL PLATFORM

A. SCARAB

The experimental platform utilized in this work, SCARAB, was designed to be able to run dynamically on

level, vertical, and inclined surfaces. A study examining the climbing performance has already been performed, in which the platform climbed at speeds up to 20cm s^{-1} while demonstrating similar dynamic behavior to the Full-Goldman climbing template [19]. This initial prototype consisted of a planar body with four legs and an electronics package. The fully assembled platform, shown in Fig. 1A, was approximately 50cm long and 30cm wide and has a mass of 1.88kg .

Each leg was rigidly attached to the body in a planar configuration and at a fixed angle. The legs had a single actuated degree of freedom driven by a brushed DC motor. The motor was connected to a crank-slider mechanism that modulated the rest length of the leg. A spring mechanism was set in series with the drive mechanism, which served to reduce peak ground reaction forces and motor loads, as well as to allow for gradual loading on the foot following attachment. The spring mechanism, shown in Fig. 1B, allowed the leg to exhibit different effective stiffness when loaded in extension or compression. Attachment to the vertical climbing substrate was achieved utilizing a hook-and-loop attachment, as used in [4]. The mechanism consisted of a debarbed fish hook that had been bent to optimally engage a Berber carpet surface.

For control of SCARAB, a custom electronics package was developed around the Gumstix Overo[®] Fire. The electronics package enabled the utilization of two dual quadrature decoders and four single channel motor drivers to individually track and control the motors, while having additional GPIOs accessible for future expansion. Motor control is currently clock-based, with each motor being independently driven at a fixed frequency.

Additional details on the physical and electrical configuration of the platform can be found in [19].

B. Modifications for Level Ground Running

While the original design of SCARAB could successfully climb vertical surfaces at high speeds, its mobility on level ground was limited. To improve its ability to run on all inclines, several design modifications were made, including the adoption of a non-planar body, the addition of compliant hips, and the modification of the attachment mechanism.

The first problem identified with the original version of SCARAB was that the body and legs dragged along the ground when on level or mildly-inclined surfaces. This resulted in significant friction and hampered engagement of the claws with the running surface. To elevate the platform, a non-planar body was adopted on which the leg mounts were bent inward to provide splay, as shown in Fig. 3A. The splay angle Ψ is defined by the angle between the projection of the leg axis on the transverse plane and the vertical axis. For example, a body with $\Psi = 90^\circ$ would have a planar configuration while a body with $\Psi = 0^\circ$ would be fully upright. This modification results in the claws moving towards or away from the ground during extension and retraction, respectively. This allows each leg to have a flight phase, during which the leg is able to move forward without the claws catching on the running surface. For the subsequently described experiments, a splay angle of 50°

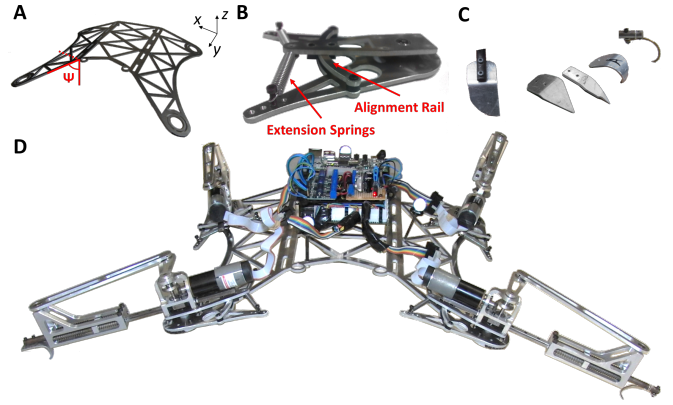


Fig. 3. (A) Non-planar SCARAB body. This modified body incorporates a splay angle Ψ , measured between the vertical and transverse axes, to allow the legs to lift off the ground and reset to their nominal position. (B) Compliant hip mechanism that allows the legs to rotate when the leg is in stance and reset during flight. (C) Front foot attachment mechanisms. These are several prototype claws tested for use on the front legs for running. The curved claw was utilized in the final version as it gave sufficient attachment during stance while being able to slide along the running surface to reset without catching during flight. (D) Updated SCARAB platform in its running configuration.

was utilized, as this provided a sufficient angle to disengage the claws while still keeping the legs at an angle that allowed for significant lateral force generation.

An additional issue with the original design was that the rigid attachment of the hips to the body limited to the front legs to act as pure breaking elements. By modifying the hips to allow rotation, the legs could switch from breaking to accelerating as they rotated during stance. Furthermore, dynamic climbing simulations indicated that the addition of hip compliance would improve vertical locomotion as well. A mechanism was developed that would allow in-plane rotation of the hips, shown in Fig. 3B. A pin joint was used at the hip to allow rotation while antagonistic extension springs drove the leg towards the desired touch-down angle. An additional alignment slider was added to restrict out-of-plane motion of the leg.

The final platform modification was the replacement of the front and rear claws and alteration of the running surface. The need to change the attachment mechanism resulted from the front claws engaging in compression during running; however, rotating the front claws caused them to dig in to the running surface and never disengage. To remedy this, bent aluminum plates with a curved front edge, shown in Fig. 3C, were utilized that engaged the surface through frictional attachment but could easily be lifted from the surface. A rubberized tile was also used instead of carpet to keep the claws from catching when disengaged. Debarbed fish hooks were still used for the rear claws, though the angle of incidence with the surface was made much steeper to engage the rubber surface.

The resulting platform after modifications is shown in Fig. 3D. With the addition of compliant hips, SCARAB still demonstrates effective dynamic climbing, and when using a splay angle of 70° or larger, it is able to climb

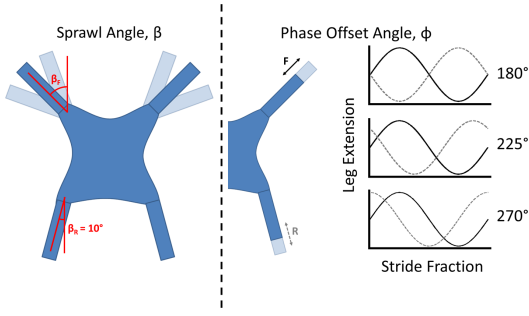


Fig. 4. (A) Sprawl angle is determined by measuring the angle between the projection of the leg axis on the coronal plane and the central body axis. (B) Phase offset angle is determined by the phase difference in the front and rear extensions on the same side of the body. Legs on the opposite side of the body are mirror by 180° .

as well; however, modifications to the claws results in failed attachment on surfaces with inclines greater than 30° . Future platform developments will consider the addition of adjustable splay angles and examine alternative claws to allow SCARAB to utilize both locomotion modalities without manual alterations.

IV. EXPERIMENTAL PROCEDURE

A. Parameter Study

Numerous parameters affect the locomotion performance of SCARAB. These include physical parameters, such as leg stiffness and length, configuration parameters, such as sprawl and splay angle, and control parameters, such as actuation speed and leg phasing. Since the physical parameters of the platform were established via dynamic scaling [4] and their variation has been previously investigated [24], they were omitted from this study. The study parameters were chosen on the basis of the anticipated effect on performance, the ease at which they could be controlled, and their potential for on-the-fly adaptation in future platform iterations. This led to the selection of two parameters to be examined in this analysis: front leg sprawl angle β and phase offset angle Φ .

The first parameter, front leg sprawl angle, is the angle between the projection of the front leg axis on the coronal plane and the central body axis, as shown in Fig. 4A. The front leg sprawl angle is anticipated to have a significant effect since it directly impacts the direction of the initial braking force in the front legs. Adjustment of the front leg sprawl angle on the current platform is performed by manually adjusting the angle at which the compliant hip mechanism is attached to the body.

The second parameter, phase offset angle, shown in Fig. 4B, is the phase difference between the leg extension of the front and rear legs on the same side of the body. Legs on opposite sides of the body were kept 180° out-of-phase for all trials, limiting the examination to symmetric gaits. The phase offset angle is expected to have an impact due to the importance of gait selection at a given actuation speed. While many platforms arbitrarily selected gaits, animals utilize a number of different gaits and often switch depending on their speed [25]. The adjustment of this parameter is straight

forward, as the offset can be prescribed directly via the motor controller.

The range of the parameter variations was determined in preliminary studies. For front leg sprawl angles of 90° and greater, the front legs were unable to brake, resulting in the robot tipping over. Additionally, when the front sprawl angle was less than 45° , the front legs could only brake and the claws tended to catch, causing irregular behavior. This prompted the selection of the range of sprawl angles to be from 40° to 80° in 10° increments. For the phase offset angle, initial tests examined a range of offsets from 90° to 270° . For phase offsets of less than 180° , the robot could not maintain a heading and would rapidly veer off-course. This led to three phase offsets to be examined in the experimental parameter study: 180° , 225° , and 270° .

B. Data Acquisition

To obtain SCARAB performance data, trials were run for each of the 15 configurations (5 sprawl angles and 3 phase angles). Center of mass trajectory data was captured using a two-camera Vicon Bonita motion tracking system. This system allowed for three dimensional position and orientation data to be recorded at 100Hz . For each configuration of the robotic platform, 10 trials were run, with SCARAB running for approximately 25 strides over the course of each trial. From each trial, 6 consecutive strides were selected as a representative data set for the run. This provided sufficient time for the system to reach steady state while ensuring that the data set is large enough to rule out bias in selecting the strides.

C. Data Analysis

The data acquired via the Vicon system contained position and orientation data for each stride. To account for differences in the initial heading of the platform, the data was first reoriented such that lateral and vertical displacement over the course of each stride was set to zero. To calculate velocity, a difference formula was used after applying a 3rd order moving average filter using MATLAB's *filter* function to the position data.

SCARAB's running performance through the experimental trials was quantified via two methods. First, the mean fore-aft velocity over the course of a stride was compared for each set of sprawl angles and phase offset angles. Second, mean horizontal and fore-aft velocities profiles for the platform were compared to those for the LLS template. Characteristics examined for similarity include the profile shape, the phasing of the fore-aft and lateral velocities, and the relative maxima and variation in the velocities.

V. RESULTS AND DISCUSSION

To compare the effects of front leg sprawl angle and phase offset angle on running speed, the mean fore-aft velocity was examined from the experimental data for each parameter configuration and is plotted in Fig. 5. From this data, we observe, first, that over the range of front leg sprawl angles examined, the use of a 180° phase offset resulted in reduced

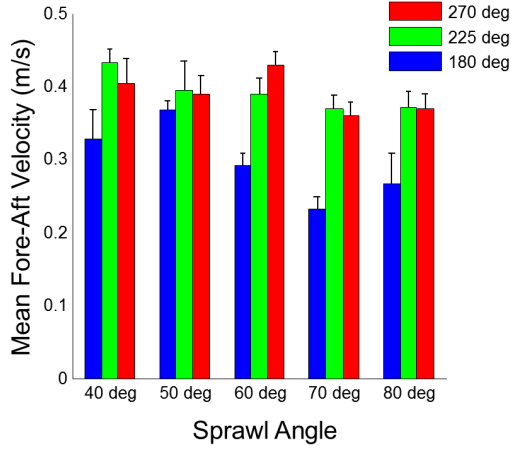


Fig. 5. SCARAB running speed as a function of sprawl angle and phase offset angle. The bars show the mean fore-aft velocity each sprawl angle/phase offset parameter set. The error bars show the standard deviation in mean fore-aft velocity for all trials with a given parameter set.

fore-aft velocities as compared 225° or a 270° phase offsets with the same sprawl angle. Furthermore, during the 180° phase offset trials, it was observed that the platform exhibited erratic motions, violently shaking the platform, which were not noted for other phase angles. Second, use of a front leg sprawl angle between 40° and 60° resulted in the highest fore-aft velocities, demonstrating mean running speeds around 40cms^{-1} . While the phase offset angle resulting in the highest mean fore-aft velocity varies with sprawl angle, these differences are not statistically significant.

After considering the speeds of the different platform configurations, the velocity profiles were examined. These profiles were compared to the biologically-inspired LLS template, shown in the first column of Fig. 6, to determine the degree to which SCARAB was able to demonstrate similar dynamics to the model. The profiles for the LLS template were adapted from [24], for which the parameter set was chosen to match the horizontal plane dynamics of cockroaches. Five characteristics of the LLS velocity profiles are compared to the robot velocity profiles to ascertain how well the dynamics corresponded.

The first of these characteristics is the number of peaks per stride for the fore-aft and lateral velocities. For the LLS model, two fore-aft velocity peaks and one lateral velocity peak occur every stride. The second characteristic is the phasing of the fore-aft and lateral velocities. For the LLS model, the fore-aft velocity peaks are coincident with the maximum positive and negative lateral velocities while the fore-aft velocity minimums correspond to the lateral velocity being zero. The third characteristic is the relative magnitude of the lateral velocity, which is measured as $v_{lat\ peak}/v_{fa\ mean}$, where $v_{lat\ peak}$ is the maximum magnitude of the lateral velocity and $v_{fa\ mean}$ is the mean fore-aft velocity. For the LLS model, this ratio is 10%. The fourth characteristic is the percent of variation in the fore-aft velocity, calculated as $\frac{v_{fa\ max} - v_{fa\ min}}{v_{fa\ mean}}$, where $v_{fa\ max}$ and $v_{fa\ min}$ are the maximum

TABLE I
CHARACTERISTICS OF THE VELOCITY PROFILES

| Profile Characteristic | LLS | $\Phi = 180^\circ$ | $\Phi = 225^\circ$ | $\Phi = 270^\circ$ |
|-------------------------------|------|--------------------|--------------------|--------------------|
| Peaks per Stride [†] | 2/1 | 1/1 | 2/1 | 2/1 |
| Velocity Phasing | 0° | — | 30° | 10° |
| Relative Lat Velocity | 10% | 125% | 116% | 98% |
| F-A Velocity Var. | 2.5% | 63% | 38% | 22% |
| Lat Velocity Ratio | 1 | 1.83 | 1.15 | 1.02 |

[†] - Fore-aft and lateral peaks per stride are denoted on the left and right side of the slash, respectively.

and minimum fore-aft velocities, respectively. For the LLS model, this value is 2.5%. The final characteristic considered is the ratio of maximum positive and negative lateral velocities. For the LLS model, they are equal, resulting in a ratio of 1.

The SCARAB velocity profiles were examined for all sprawl angle and phase offset angle combinations; however, as the velocity profile behavior was fairly invariant to sprawl angle, we only present the results for $\beta = 60^\circ$ here. This sprawl angle was selected for detailed analysis because it was at the midpoint of the examined front sprawl angles and showed some of the fastest speeds. The SCARAB profiles with this sprawl angle and phase offsets of $\Phi = 180^\circ$, $\Phi = 225^\circ$ and $\Phi = 270^\circ$ are shown in the final three columns of Fig. 6 and the characteristics of these profiles are reported in Table I.

Both qualitative examination of the velocity profiles and consideration of the profile characteristics show that SCARAB's dynamics are quite distinct to the LLS model when using a phase offset angle of 180° or 225°. The former is dissimilar in almost every characteristic, with the only similarity being the presence of a single lateral peak per stride. The latter shows the same number of peaks and has a relatively symmetric gait, but does not match the relative magnitudes or phasing well.

Conversely, the velocity profiles and characteristics for $\Phi = 270^\circ$ are quite similar to the LLS model. The number of peaks, velocity phasing, and lateral velocity ratio agrees with the qualitative observation that with this phase offset angle, the robot velocity profiles have a similar shape to the LLS model. The relative lateral velocity and variation in fore-aft velocity are still significantly different, though both are much closer to the model's characteristics than with the other phase offset angles.

With only this coarse parameter sweep, we can conclude that with $\beta = 60^\circ$ and $\Phi = 270^\circ$, SCARAB is able to run at its fastest speeds with similar dynamics to the LLS model. The two most significant discrepancies between the robot and the model are the large values for relative magnitude of lateral velocity and variation in fore-aft velocity on the platform. The increased variation in fore-aft velocity is in part a result of not fully optimizing the robot parameters. The presence of larger than expected lateral velocities may also have contributed to the high degree of variation in the fore-aft velocity.

It is notable that the relative lateral velocity magnitude

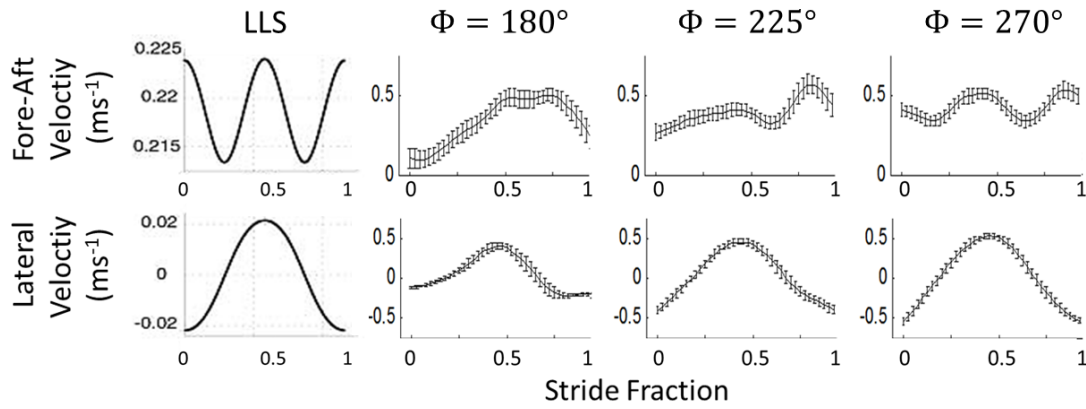


Fig. 6. Fore-aft and lateral velocity profiles for the LLS model and the SCARAB platform. The first column shows the LLS velocity profiles, which were adapted from [26]. The second, third and fourth columns show the experimental velocity and force profiles for $\Phi = 180^\circ$, $\Phi = 225^\circ$ and $\Phi = 270^\circ$, respectively. For each of these results, $\beta = 60^\circ$. The error bars in the final three columns illustrate the standard deviation at each fractional step of a stride.

is consistently large across all phase offset angles. The platform's splay angle is likely the cause of this discrepancy. A splay angle of 50° was selected to emphasize the horizontal plane motion, but the experimental results indicate that such a large splay angle may be undesirable. This suggests that a more upright posture for SCARAB will improve the similarity to the LLS model.

VI. CONCLUSIONS

In this work, we investigate and analyze the ability of SCARAB to run dynamically on level ground. While other robots have demonstrated periodic lateral dynamics [27], [28], these experiments show that SCARAB is the first to effectively utilize the horizontal plane dynamics in a manner quantifiably similar to the LLS model. Additionally, as we have previously established the platform's ability to dynamically climb on vertical surfaces, these results show that SCARAB is the first platform to demonstrate multiple *dynamical* locomotion modalities. We present several modifications that were incorporated into SCARAB to allow it to exhibit this running behavior and performed an analysis of front leg sprawl angle and leg phasing to determine the most effective configuration for rapid locomotion on level ground. In addition to showing locomotion characteristics similar to the LLS model, we found that using the set of parameters that demonstrates the best correlation to the LLS dynamics also results in the fastest running speeds.

Future developments with the SCARAB platform will aim to augment our understanding of the locomotion dynamics that lead to high-speed and robust locomotion on level, inclined, and vertical surfaces. Our analyses will be extended to examine the sagittal plane dynamics and studies on inclines will be performed to examine the transitional regime as the robot moves from running to climbing. Physical modifications to the platform will also be performed. As already discussed in this work, adjusting the splay angle to provide the platform with a more upright posture is expected to result in greater similarity to the LLS dynamics. Also, since the platform currently requires manual interference to switch

locomotion modalities, we will examine the development of novel mechanisms to adjust system parameters on the fly. This will include methods of adjusting splay and sprawl angles as well as multipurpose feet capable of engaging both running and climbing substrates.

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